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LETTER TO THE EDITOR

A fully interacting many-electron–phonon system in one dimension: an exactly soluble model

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Abstract

A one-dimensional electron gas problem is investigated within the framework of the Luttinger model incorporating both the electron–acoustic-phonon interaction and the electron–optical-phonon interaction together with the electron–electron interaction. It is shown that this model is exactly soluble and the energy spectrum is constituted by bosonic excitations. It is also shown that for certain values of the coupling parameters the system exhibits an instability indicating the possibility of a phase transition.

With the recent advances in materials fabrication techniques, it is now possible to realize high-quality quasi-one-dimensional structures which are called quantum wires. The quantum wire structures have many interesting and fascinating properties, and have tremendous potentiality for applications in ultrafast microelectronic device technology which has motivated in recent times both theorists and experimentalists to investigate various properties of quasi-one-dimensional and purely one-dimensional electron gas systems. In the past, however, the existence of several quasi-one-dimensional conducting systems and the possibility of high-temperature superconductivity in some of them provided substantial impetus to the development of theoretical research in the field of one-dimensional systems. Indeed, there has been considerable progress in the field of the one-dimensional electron gas problem in the last few decades because of the success of the idea of bosonization which was introduced by Tomonaga [1] in his celebrated model for the one-dimensional electron system which is exactly soluble. A somewhat better model was later proposed by Luttinger [2] who introduced new states as compared to the Tomonaga model, which are filled up in the ground state, and this model is also exactly soluble. Both the Tomonaga model and the Luttinger model, however, neglect lattice excitations such as phonons and their interactions with electrons. Electron–phonon interactions play in general an important role in transport phenomena and are also relevant in polar semiconductors where the natural quasiparticles are

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polarons (see [3] for a review on polarons). Several investigations (see [4] and references therein) have revealed that the effects of electron–phonon interactions become more and more pronounced with the reduction in the dimensionality of the systems and therefore the static and dynamic properties of electrons confined in quantum wire structures are expected to be strongly modified because of the interaction with the phonon modes of these systems. Indeed, there have been several investigations on the polaron gas problem in a quantum wire using various many-body techniques and a great deal of literature has already piled up with interesting results. Stroscio [5] has generalized the Fröhlich Hamiltonian for a one-dimensional quantum wire with rectangular cross-section and has calculated the total scattering rate for electron–longitudinal-optical-phonon interaction considering the case where phonon modes are confined in two of the three spatial directions. Vasilopoulos *et al* [6] have shown that magneto-conductivity in quasi-one-dimensional wires decreases when the confinement becomes stronger. Degani [7] has investigated the energy–momentum relations for electrons confined in rectangular quantum wires of GaAs surrounded by GaAlAs using an improved Brillouin–Wigner perturbation theory and has observed a significant energy lowering when the electron momentum approaches the LO-phonon momentum, and this bending over of the dispersion curve increases when the confinement is enhanced. Campos, Degani and Hipolito [8] have shown using a self-consistent approximation [9] that electron–electron and electron–phonon interactions are more effective in quantum wires of small sizes and also the correlation and exchange effects are evident in wires of reduced dimensions. Hai, Peeters and Devreese [10] have calculated the ground-state energy of the polaron gas with different quantum wire structures using the Hartree–Fock method and the random-phase approximation, and have shown that screening due to electron gas reduces the electron–phonon coupling. In all these investigations, however, the conduction band of the electron in the absence of any interaction has been taken to be parabolic. Marino [11] has considered the problem of a one-dimensional electron gas incorporating the effects of the electron–acoustic-phonon interaction and the electron–electron interaction within the framework of the Luttinger model. He has shown that this problem is exactly soluble and the spectrum contains only gapless excitations, and so the system exhibits a metallic behaviour. For certain values of the coupling constants, however, the system has been shown to develop a polaronic superconductivity. Later, Martins [12] studied the effect of electron–optical-phonon interaction and the electron–acoustic-phonon interaction within the framework of the same model but has neglected the electron–electron interaction.

In the present letter we consider an interacting electron–phonon system in one dimension incorporating both the electron–acoustic-phonon interaction and the electron–optical-phonon interaction together with the electron–electron interaction within the framework of the Luttinger model, and show that, even for this more complete model, it is possible to decouple the electron and the phonon fields exactly and obtain a Hamiltonian with an effective electron–electron interaction which can be written in terms of bosonic field operators. Diagonalization of this Hamiltonian using a unitary transformation then yields an exact energy spectrum for our fully interacting one-dimensional electron–phonon system

The total Hamiltonian for an interacting electron–phonon system in one dimension can be written as

$$H = H_{el} + H_{ph}^{op} + H_{ph}^{ac} + H_{ep}^{op} + H_{ep}^{ac} + H_{e-e} \tag{1}$$

where H_{el} is the free-electron Hamiltonian, H_{ph}^{op} is the optical-phonon Hamiltonian, H_{ph}^{ac} is the acoustic-phonon Hamiltonian, H_{ep}^{op} represents the electron–optical-phonon Hamiltonian, H_{ep}^{ac} refers to the electron–acoustic-phonon Hamiltonian and H_{e-e} is the electron–electron

interaction. The free-electron Hamiltonian H_{el} for the Luttinger model can be written as

$$H_{el} = \sum_k k v_F (a_{1k}^\dagger a_{1k} - a_{2k}^\dagger a_{2k}) \tag{2}$$

where v_F is the Fermi velocity, $a_{1k}^\dagger$ (a_{1k}) is the creation (annihilation) operator for a type-1 fermion with wave vector k and energy $\epsilon_k = k v_F$ and $a_{2k}^\dagger$ (a_{2k}) is the creation (annihilation) operator for a type-2 fermion with a wave vector k and energy $\epsilon_k = -k v_F$. The two types of fermion are quite independent, so we have the canonical anti-commutation relations, $a_{i,k,s}, a_{j,k',s'}^\dagger = \delta_{ij}\delta_{kk'}\delta_{ss'}$, $i, j = 1, 2$. The optical-phonon Hamiltonian can be written as

$$H_{ph}^{op} = \sum_k \omega_0 b_k^\dagger b_k \tag{3}$$

where $b_k^\dagger$ (b_k) is the creation (annihilation) operator for an optical phonon of frequency ω_0 and we have chosen units in which $\hbar = 1$. The acoustic-phonon Hamiltonian is given by

$$H_{ph}^{ac} = \sum_k v_s |k| \tilde{b}_k^\dagger \tilde{b}_k \tag{4}$$

where $\tilde{b}_k^\dagger$ ($\tilde{b}_k$) is the creation (annihilation) operator for an acoustic phonon and v_s is the sound velocity. The electron–optical-phonon interaction for the present one-dimensional problem can be written as

$$H_{ep}^{op} = \frac{g}{\sqrt{2L}} \sum_k \sqrt{\omega_0}[\rho_1(k) + \rho_2(k)](b_k + b_{-k}^\dagger) \tag{5}$$

where L is the length of the system, g is the electron–optical-phonon coupling constant and $\rho_i(k)$ is the electron-density operator given by $\rho_i(k) = \sum_q a_{i,q+k}^\dagger a_{i,q}$, $i = 1, 2$. The electron–acoustic-phonon Hamiltonian is given by

$$H_{ep}^{ac} = \frac{\lambda \mathrm{i}}{\sqrt{2L}} \sum_{k>0} \sqrt{|k| v_s}[\rho_1(k) + \rho_2(k)](\tilde{b}_k + \tilde{b}_{-k}^\dagger) + \text{H.C.} \tag{6}$$

where λ is the electron–acoustic-phonon coupling constant. Before we consider the form of electron–electron interaction, we shall introduce the real-space field operators:

$$\psi_i(x) = \frac{1}{\sqrt{L}} \sum_k \mathrm{e}^{\mathrm{i}kx} a_{ik} \tag{7}$$

$$\phi(x) = \frac{1}{\sqrt{L}} \sum_k \frac{1}{\sqrt{2\omega_0}} (b_k + b_{-k}^\dagger) \mathrm{e}^{\mathrm{i}kx} \tag{8}$$

$$\tilde{\phi}(x) = \frac{1}{\sqrt{L}} \sum_k \frac{1}{\sqrt{2v_s|k|}} (\tilde{b}_k + \tilde{b}_{-k}^\dagger) \mathrm{e}^{\mathrm{i}kx}. \tag{9}$$

The total Hamiltonian in terms of the field operators is given by

$$\begin{aligned} H = \int \mathcal{H}\, \mathrm{d}x = {} & -\mathrm{i}v_F \int \Psi^\dagger(x)\sigma_3\, \partial_x \Psi(x)\, \mathrm{d}x + \frac{1}{2}\int [\dot{\phi}^2(x) + \omega_0^2 \phi^2(x)]\, \mathrm{d}x \\ & + \frac{1}{2}\int [\dot{\tilde{\phi}}^2(x) + v_s^2 (\partial_x \tilde{\phi}(x))^2]\, \mathrm{d}x + g\omega_0 \int \Psi^\dagger(x)\Psi(x)\phi(x)\, \mathrm{d}x \\ & + \lambda v_s \int \Psi^\dagger(x)\Psi(x)\, \partial_x \tilde{\phi}(x)\, \mathrm{d}x \\ & + \frac{1}{2}\int \Psi^\dagger(x)\Psi^\dagger(y) v(x-y)\Psi(y)\Psi(x)\, \mathrm{d}x\, \mathrm{d}y \end{aligned} \tag{10}$$

where $\mathcal{H}$ is the Hamiltonian density, $\Psi(x)$ is the two-component field operator given by

$$\Psi = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix}$$

and σ_3 is a Pauli matrix, and the last term refers to the electron–electron interaction with $v(x-y)$ representing the electron–electron interaction potential which is a function of $|x-y|$. In order to describe the dynamics of the system in terms of polarons, one has to eliminate the phonon fields from the theory. This can be conveniently achieved by introducing the Lagrangian density L corresponding to $\mathcal{H}$. We choose the Lagrangian density as

$$\begin{aligned} L = {} & \mathrm{i}\Psi^\dagger(x)\,\partial_t\Psi(x) + \mathrm{i}v_F\Psi^\dagger(x)\sigma_3\,\partial_x\Psi(x) + \frac{1}{2}[\dot{\phi}^2(x) - \omega_0^2\phi^2(x)] \\ & - g\omega_0\Psi^\dagger(x)\Psi(x)\phi(x) - \lambda v_s\Psi^\dagger(x)\Psi(x)\,\partial_x\tilde{\phi}(x) \\ & + \frac{1}{2}[\dot{\tilde{\phi}}^2(x) - v_s^2(\partial_x\tilde{\phi}(x))^2] + \dot{\phi}\chi \\ & + \dot{\tilde{\phi}}\tilde{\chi} - \frac{1}{2}\int \Psi^\dagger(x)\Psi^\dagger(y)v(x-y)\Psi(y)\Psi(y)\,\mathrm{d}x\,\mathrm{d}y \end{aligned} \tag{11}$$

where we have introduced two additional terms $\dot{\phi}\chi$ and $\dot{\tilde{\phi}}\tilde{\chi}$ which however do not affect the equation of motion and the Hamiltonian density. We have to choose χ and $\tilde{\chi}$ appropriately to eliminate the phonon fields. Choosing $\chi = -\dot{\phi}$ and $\tilde{\chi} = -\dot{\tilde{\phi}}$ we get from Lagrange's equations of motion

$$\phi = -\frac{g}{\omega_0}\Psi^\dagger(x)\Psi(x) \qquad \partial_x\tilde{\phi} = -\frac{\lambda}{v_s}\Psi^\dagger(x)\Psi(x). \tag{12}$$

To make further progress we observe that L is invariant under the transformations $\Psi(x) \to \Psi^\dagger(x) = \mathrm{e}^{\mathrm{i}\theta}\psi(x)$ and $\Psi(x) \to \Psi^\dagger(x) = \mathrm{e}^{\mathrm{i}\sigma_3\theta}\psi(x)$ which immediately give two conservation laws. These conservation equations together with equation (12) finally give

$$\ddot{\phi} - v_F^2\,\partial_x^2\phi = 0 \qquad \ddot{\tilde{\phi}} - v_F^2\,\frac{\partial^2}{\partial x^2}\tilde{\phi} = 0 \tag{13}$$

which imply that the two phonon fields are completely decoupled. The total Hamiltonian can now be written as

$$\begin{aligned} H = H_{el} + H_{e-e}^{eff} = {} & \sum_k k v_F(a_{1k}^\dagger a_{1k} - a_{2k}^\dagger a_{2k}) \\ & + \sum_k \left[\frac{1}{2L}\left\{2g^2\left(\frac{v_F^2k^2}{\omega_0^2} - 1\right)\right\} + \frac{\lambda^2}{L}\left(\frac{v_F^2}{v_s^2} - 1\right) + \frac{v_{1k}}{2L}\right] \\ & \times [\rho_1(k)\rho_1(-k) + \rho_2(-k)\rho_2(k)] \\ & - \sum_{\text{all } k}\left[\frac{1}{2L}\left\{2g^2\left(\frac{v_F^2k^2}{\omega_0^2} + 1\right)\right\} - \frac{\lambda^2}{L}\left(\frac{v_F^2}{v_s^2} + 1\right) + \frac{v_{1k}}{2L}\right]\rho_1(k)\rho_2(-k) \end{aligned} \tag{14}$$

where $v_{1q} = v_q + v_{-q}$ with

$$v_q = \int\int v(x-y)\mathrm{e}^{\mathrm{i}q(x-y)}\,\mathrm{d}x\,\mathrm{d}y.$$

One can identify the density operators $\rho_i(k)$ as bosonic operators from their commutation relations:

$$[\rho_i(-p), \rho_j(-p')] = \delta_{ij}\delta_{pp'}\frac{pL}{2\pi} \qquad p, p' > 0 \tag{15}$$

and write down the total Hamiltonian as

$$H = \sum_{k>0}\left[\frac{1}{2L}\left\{2g^2\left(\frac{v_F^2k^2}{\omega_0^2}-1\right)+4\pi v_F\right\}+\frac{\lambda^2}{L}\left(\frac{v_F^2}{v_s^2}-1\right)+\frac{v_{1k}}{2L}\right]$$
$$\times\left[\rho_1(k)\rho_2(-k)+\rho_2(-k)\rho_1(k)\right]$$
$$-\sum_{\text{all}\,k}\left[\frac{1}{2L}\left\{2g^2\left(\frac{v_F^2k^2}{\omega_0^2}+1\right)\right\}-\frac{\lambda^2}{L}\left(\frac{v_F^2}{v_s^2}+1\right)-\frac{v_{1k}}{2L}\right]\rho_1(k)\rho_2(k) \tag{16}$$

where H_{el} has been replaced as usual by the operator

$$\frac{2\pi}{L}v_F\sum_{k>0}\left\{\rho_1(k)\rho_1(-k)+\rho_2(-k)\rho_2(k)\right\} \tag{17}$$

which satisfies the same commutation relations with ρ as H_{el} does. The effective Hamiltonian equation (16) can now be diagonalized in the usual way by the unitary transformation with an operator

$$\exp\left[\mathrm{i}\frac{2\pi\,\mathrm{i}}{L}\sum_{\text{all}\,k}\frac{\Phi(k)}{k}\rho_1(k)\rho_2(-k)\right]$$

with

$$\Phi(k)=\frac{1}{2}\tanh^{-1}\left(-\frac{D_2(k)}{D_1(k)}\right)$$

where

$$D_1(k)=\left[\frac{1}{2L}\left\{2g^2\left(\frac{v_F^2k^2}{\omega_0^2}-1\right)+4\pi v_F\right\}+\frac{\lambda^2}{L}\left(\frac{v_F^2}{v_s^2}-1\right)+\frac{v_{1k}}{2L}\right] \tag{18}$$

and

$$D_2(k)=\frac{1}{2L}\left\{2g^2\left(\frac{v_F^2k^2}{\omega_0^2}+1\right)\right\}-\frac{\lambda^2}{L}\left(\frac{v_F^2}{v_s^2}+1\right)-\frac{v_{1k}}{2L} \tag{19}$$

and the energy spectrum of the system can be described exactly in terms of bosonic excitations with the dispersion relation

$$\varepsilon(k)=v_F|k|\left(1-\frac{g^2+\lambda^2}{\pi v_F}+\frac{v_{1k}}{2\pi v_F}\right)^{1/2}\left(1+\frac{g^2k^2v_F}{\pi\omega_0^2}+\frac{\lambda^2v_F}{\pi v_s^2}\right)^{1/2}. \tag{20}$$

The above bosonization and the diagonalization procedure is however valid only if

$$\frac{(g^2+\lambda^2)}{\pi v_F}<\left(1+\frac{v_{1k}}{2\pi v_F}\right).$$

Thus, for a certain range of the coupling parameters the spectrum of a one-dimensional electron–phonon system would be gapless and one would then essentially have a polaronic metal. The system however has an instability, indicating the possibility of a phase transition if the coupling constants satisfy the condition

$$g_c^2+\lambda_c^2=\pi v_F+\frac{1}{2}v_{1k}. \tag{21}$$

To determine the nature of this instability and the type of the associated phase transition, one has however to calculate various possible correlation functions for the system.

In conclusion, we have studied a fully interacting one-dimensional electron–phonon system within the framework of the Luttinger model incorporating both the electron–acoustic-phonon and electron–optical-phonon interaction together with the electron–electron interaction

and have shown that the acoustic- and the optical-phonon fields can be eliminated from the problem yielding an exact effective Hamiltonian which can be written completely in terms of bosonic operators and can be diagonalized by a unitary transformation to yield an exact excitation spectrum which is gapless. For certain values of the coupling parameters the system exhibits an instability indicating the possibility of a phase transition. The detailed nature and the type of the instability can be predicted from the behaviour of various correlation functions possible for the system.

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